Power Network Dynamics & Control

LANL Grid Science School

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Why care about power system dynamics & control?

www.offthegridnews.com

1. increasing renewables & deregulation
2. growing demand & operation at capacity

⇒ increasing volatility & complexity, decreasing robustness margins

Rapid technological and scientific advances:

1. re-instrumentation: sensors & actuators
2. complex & cyber-physical systems

⇒ cyber-coordination layer for smart grid

⇒ need to understand the **complex** network dynamics & control

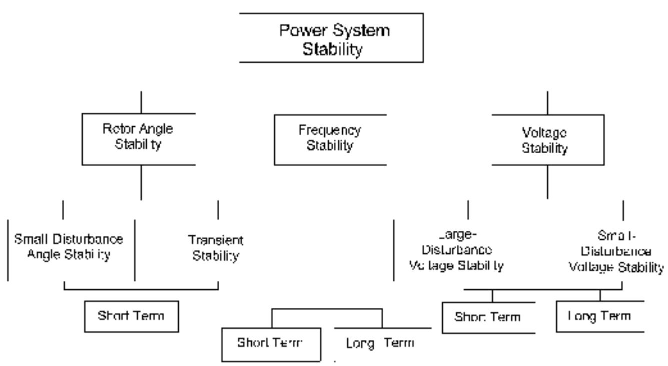
One system with many dynamics & control problems

IEEE TRANSACTIONS ON POWER SYSTEMS, VOL. 19, NO. 2, MAY 2004 1387

Definition and Classification of Power System Stability

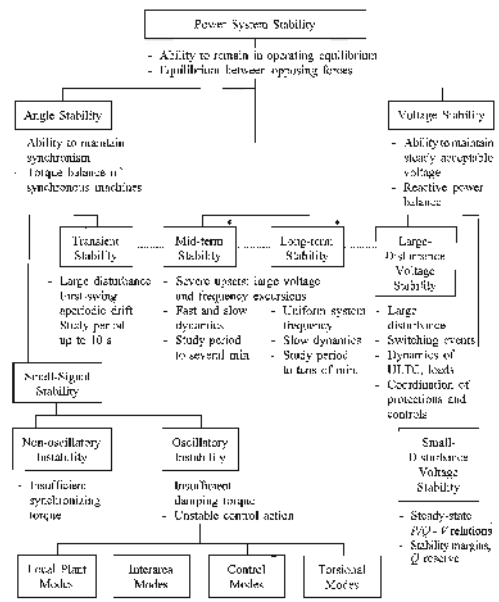
IEEE/CIGRE Joint Task Force on Stability Terms and Definitions

Prabha Kundur (Canada, Convener), John Paserba (USA, Secretary), Venkat Ajjarapu (USA), Göran Andersson (Switzerland), Anjan Bose (USA), Claudio Canizares (Canada), Nikos Hatziargyriou (Greece), David Hill (Australia), Alex Stankovic (USA), Carson Taylor (USA), Thierry Van Cutsem (Belgium), and Vijay Vittal (USA)



We have to make a choice based on . . .

many aspects depending on spatial/temporal/state scales



- what future speakers need and what will be covered by others
- what I actually know well
- what is interesting from a network perspective rather than from device perspective
- what is relevant for future (smart) power grids with high renewable penetration
- what gives rise to fun distributed control problems

Outline

- Introduction
- Power Network Modeling
- Feasibility, Security, & Stability
- Power System Control Hierarchy
- Power System Oscillations
- Conclusions

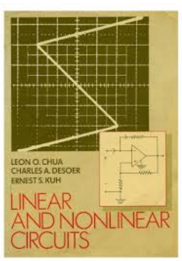
my particular focus is on **networks**

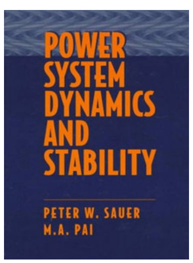
Disclaimers

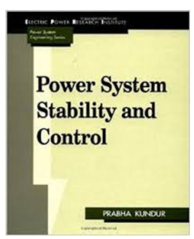
- start-off with "boring" modeling before we get to "sexy" topics
- we will cover mostly basic material & some recent "cutting edge" work
- we will focus on simple models and developing physical & math intuition
- will give references to more complex models & more recent research
- we will not go deeply into the math though everything is sound
- want to convey intuition and give references to look up the details
- notation is mostly "standard" (watch out for sign & p.u. conventions)
- ask me for further reading about any topic
- interrupt & correct me anytime

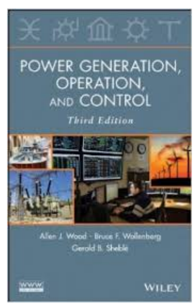
Many references available . . . my personal look-up list

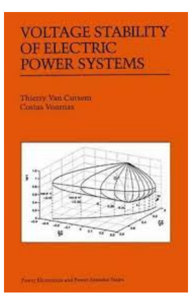
. . . to be complemented by references throughout the lecture

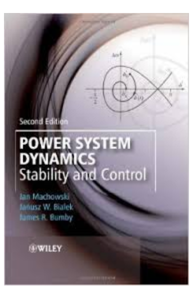


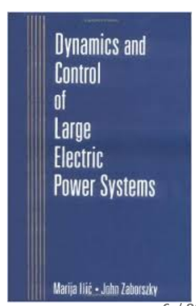












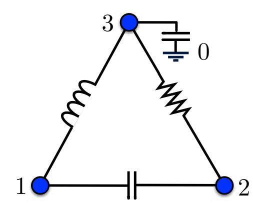
Outline

- Introduction
- **Power Network Modeling**
  - Circuit Modeling: Network, Loads, & Devices
  - Kron Reduction of Circuits
  - Power Flow Formulations & Approximations
  - Dynamic Network Component Models
- Feasibility, Security, & Stability
- Power System Control Hierarchy
- Power System Oscillations
- Conclusions

Circuit Modeling: Network, Loads, & Devices

AC circuits – graph-theoretic modeling

1. a circuit is a connected & undirected **graph** $G = (\mathcal{V}, \mathcal{E})$
   - $\mathcal{V} = \{1, \dots, n\}$ are the nodes or *buses*
     - buses are partitioned as $\mathcal{V} = \{\text{sources}\} \cup \{\text{loads}\}$
     - the ground is sometimes explicitly modeled as node 0 or $n+1$
   - $\mathcal{E} \subset \{\{i,j\} : i,j \in \mathcal{V}\} = \mathcal{V} \times \mathcal{V}$ are the undirected edges or *branches*
     - edges between distinct nodes $\{i,j\}$ are the *lines*
     - self-edges $\{i,i\}$ (or edges to ground $\{i,0\}$) are the *shunts*



$$\mathcal{V} = \{1, 2, 3\}$$

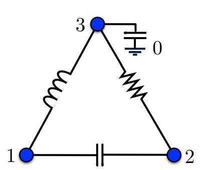
$$\mathcal{E} = \{\{1,2\}, \{1,3\}, \{2,3\}, \{3,3\}\}$$

AC circuits – the network admittance matrix

2. $Y = [Y_{ij}] \in \mathbb{C}^{n \times n}$ is the **network admittance matrix** with elements

$$Y_{ij} = \begin{cases} -\frac{1}{Z_{ij}} & \text{for off-diagonal elements } i \neq j \\ \frac{1}{Z_{i,\text{shunt}}} + \sum_{j \neq i} \frac{1}{Z_{ij}} & \text{for diagonal elements } i \neq j \end{cases}$$

- impedance = resistance + i · reactance: $Z_{ij} = R_{ij} + \mathrm{i} \cdot X_{ij}$
- admittance = conductance + i · susceptance: $\frac{1}{Z_{ij}} = G_{ij} + \mathrm{i} \cdot B_{ij}$

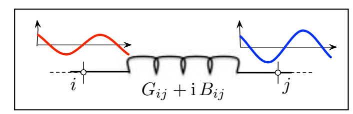


$$Y = \underbrace{\begin{bmatrix} \frac{1}{Z_{12}} + \frac{1}{Z_{13}} & -\frac{1}{Z_{12}} & -\frac{1}{Z_{13}} \\ -\frac{1}{Z_{12}} & \frac{1}{Z_{12}} + \frac{1}{Z_{23}} & -\frac{1}{Z_{23}} \\ -\frac{1}{Z_{13}} & -\frac{1}{Z_{23}} & \frac{1}{Z_{13}} + \frac{1}{Z_{23}} \end{bmatrix}}_{\text{network Laplacian matrix}} + \underbrace{\begin{bmatrix} 0 & & \\ & 0 & \\ & & \frac{1}{Z_{3,\text{shunt}}} \end{bmatrix}}_{\text{diag(shunts)}}$$

Note *quasi-stationary* modeling: $Z_{13} = \mathrm{i}\,\omega^* L_{13}$ with nominal frequency ω^*

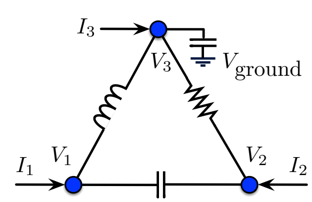
AC circuits – basic variables

3. **basic variables**: voltages & currents
   - on nodes: potentials & current injections
   - on edges: voltages & current flows



4. *quasi-stationary* **AC phasor coordinates** for harmonic waveforms:
   - e.g., complex voltage $V = E\, e^{\mathrm{i}\,\theta}$ denotes $v(t) = E \cos(\theta + \omega^* t)$

     where $V \in \mathbb{C}$, $E \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{S}^1$, $\mathrm{i} = \sqrt{-1}$, and ω^* is nominal frequency



external injections: I_1, I_2, I_3

potentials: V_1, V_2, V_3

reference: $V_{\text{ground}} = 0\text{V}$

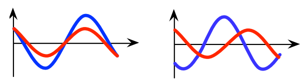
Note: quasi-stationarity assumption can be justified via singular perturbation analysis & modeling can be improved using *dynamic phasors* [A. Stankovic & T. Aydin '00].

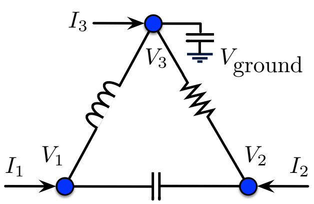
AC circuits – fundamental equations

5. **Ohm's law** at every branch: $I_{i\to j} = \frac{1}{Z_{ij}}(V_i - V_j)$
6. **Kirchhoff's current law** for every bus: $I_i + \sum_j I_{j\to i} = 0$
7. **current balance equations** (treating the ground as node with 0V):

$$I_i = -\sum_j I_{j\to i} = \sum_j \frac{1}{Z_{ij}}(V_i - V_j) = \sum_j Y_{ij} V_j \quad \text{or} \quad \boxed{I = Y \cdot V}$$

8. **complex power:** $S = V_i \bar{I}_i = P + \mathrm{i}Q$
$=$ active power $+\,\mathrm{i}\,\cdot$ reactive power



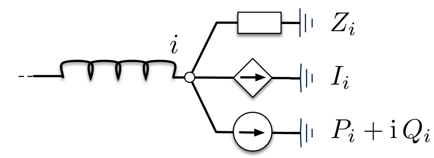


$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Note: all variables are in per unit (p.u.) scheme, i.e., normalized wrt base voltage

Static models for sources & loads

- aggregated **ZIP load model**:
  constant impedance **Z** +
  constant current **I** +
  constant power **P**



- more general **exponential load model**: power $= const. \cdot (V/V_{\text{ref}})^{const.}$
  (combinations & variations learned from data)
- conventional **synchronous generators** are typically controlled
  to have constant active power output P and voltage magnitude E
- sources interfaced with **power electronics** are typically controlled
  to have constant active power P and reactive power Q

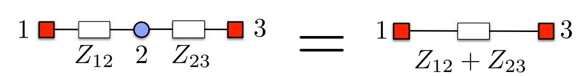
$\Rightarrow$ **PQ** buses have complex power $S = P + \mathrm{i}Q$ specified
$\Rightarrow$ **PV** buses have active power P and voltage magnitude E specified
$\Rightarrow$ **slack buses** have E and θ specified (not really existent)

Kron Reduction of Circuits

Kron reduction

[G. Kron 1939]

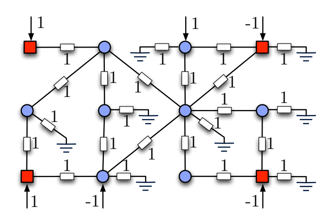
often (almost always) you will encounter Kron-reduced network models



General procedure:

0. convert const. power injections locally to shunt impedances $Z = S/V_{\text{ref}}^2$
1. partition linear current-balance equations via boundary & interior nodes:
   (arises naturally, e.g., sources & loads, measurement terminals, etc.)

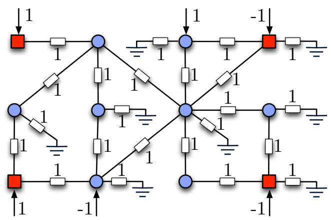
$$\begin{bmatrix} I_{\text{boundary}} \\ I_{\text{interior}} \end{bmatrix} = \left[\begin{array}{c|c} Y_{\text{boundary}} & Y_{\text{bound-int}} \\ Y_{\text{bound-int}}^T & Y_{\text{interior}} \end{array}\right] \begin{bmatrix} V_{\text{boundary}} \\ V_{\text{interior}} \end{bmatrix}$$



Kron reduction cont'd

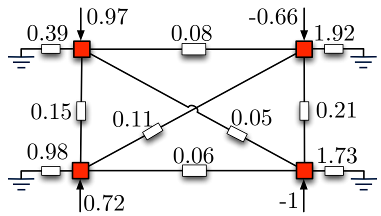
❷ Gaussian elimination of interior voltages:

$$V_{\text{interior}} = {\color{blue}Y_{\text{interior}}}^{-1} \left(I_{\text{interior}} - Y_{\text{bound-int}}^T V_{\text{boundary}} \right)$$



original circuit

$I = Y \cdot V$



"equivalent" reduced circuit

${\color{red}I_{\text{red}}} = {\color{red}Y_{\text{red}}} \cdot {\color{red}V_{\text{boundary}}}$

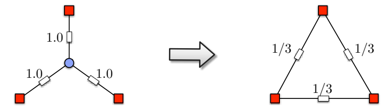
$\Rightarrow$ reduced Y-matrix: ${\color{red}Y_{\text{red}}} = {\color{red}Y_{\text{boundary}}} - Y_{\text{bound-int}} \cdot {\color{blue}Y_{\text{interior}}}^{-1} \cdot Y_{\text{bound-int}}^T$

$\Rightarrow$ reduced injections: ${\color{red}I_{\text{red}}} = I_{\text{boundary}} - Y_{\text{bound-int}} \cdot {\color{blue}Y_{\text{interior}}}^{-1} \cdot I_{\text{interior}}$

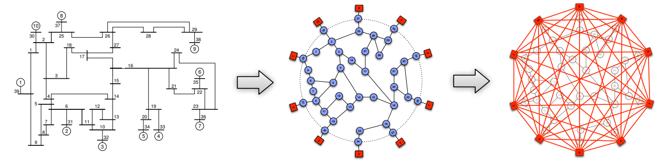
Examples of Kron reduction

algebraic properties are preserved but the network changes significantly

- Star-Δ transformation [A. E. Kennelly 1899, A. Rosen '24]



- Kron reduction of load buses in *IEEE 39 New England power grid*



$\Rightarrow$ topology without weights is meaningless!

$\Rightarrow$ shunt resistances (loads) are mapped to line conductances

$\Rightarrow$ many properties still open [FD & F. Bullo '13, S. Caliskan & P. Tabuada '14]

Power Flow Formulations & Approximations

Power balance eqn's: "power injection $= \Sigma$ power flows"

different formulations of the power flow equations

- **rectangular form:** $S_i = V_i \overline{I}_i = \sum_j V_i \overline{Y}_{ij} \overline{V}_j$ or $\boxed{S = \mathrm{diag}(V) \overline{Y V}}$

  $\Rightarrow$ purely quadratic and useful for static calculations & optimization

- **matrix form:** define unit-rank p.s.d. Hermitian matrix $W = V \cdot \overline{V}^T$ with components $W_{ij} = V_i \overline{V}_j$, then power flow is $\boxed{S_i = \sum_j \overline{Y}_{ij} W_{ij}}$

  $\Rightarrow$ linear and useful for static calculations & optimization (more later)

- **polar form:** insert $V = E e^{i\theta}$ and split real & imaginary parts:

$$\begin{aligned} \text{active power:} \quad P_i &= \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j) \\ \text{reactive power:} \quad Q_i &= -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j) \end{aligned}$$

  $\Rightarrow$ useful for dynamics, physical intuition, & system specs (today)

Power flow simplifications & approximations

power flow equations are too complex & unwieldy for analysis & large computations

- active power: $P_i \; = \; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i \; = \; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

1. **lossless** transmission lines $R_{ij}/X_{ij} = -G_{ij}/B_{ij} \approx 0$

   active power: $P_i \; = \; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j)$

   reactive power: $Q_i \; = \; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j)$

2. **decoupling** near operating point $V_i \approx 1e^{i\phi}$: $\begin{bmatrix} \partial P/\partial\theta & \partial P/\partial E \\ \partial Q/\partial\theta & \partial Q/\partial E \end{bmatrix} \approx \begin{bmatrix} \star & 0 \\ 0 & \star \end{bmatrix}$

   active power: $P_i \; = \; \sum_j B_{ij} \sin(\theta_i - \theta_j)$ (function of angles)

   reactive power: $Q_i \; = \; -\sum_j B_{ij} E_i E_j$ (function of magnitudes)

Power flow simplifications & approximations cont'd

- active power: $P_i \; = \; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i \; = \; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

3. **linearization** for small flows near operating point $V_i \approx 1e^{i\phi}$:

   active power: $P_i \; = \; \sum_j B_{ij}(\theta_i - \theta_j)$ (known as DC power flow)

   reactive power: : $Q_i \; = \; \sum_j B_{ij}(E_i - E_j)$ (formulation in p.u. system)

4. Multiple **variations & combinations** are possible
   - linearization & decoupling at arbitrary operating points
   - lines with constant R/X ratios [FD, J. Simpson-Porco, & F. Bullo '14]
   - advanced linearizations [S. Bolognani & S. Zampieri '12, '14, B. Gentile et al. '14]

Dynamic Network Component Models

Modeling the "essential" network dynamics

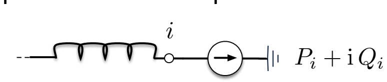
models can be arbitrarily detailed & vary on different time/spatial scales

1. active and reactive **power flow** (e.g., lossless & decoupled here)

   $P_{i,\text{inj}} = \sum_j B_{ij} \sin(\theta_i - \theta_j)$

   $Q_{i,\text{inj}} = -\sum_j B_{ij} E_i E_j$

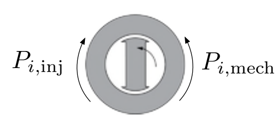
2. passive constant power **loads**



   $P_{i,\text{inj}} = P_i = const.$

   $Q_{i,\text{inj}} = Q_i = const.$

3. electromech. **swing dynamics** of synchronous machines



   $M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_{i,\text{mech}} - P_{i,\text{inj}}$

   $E_i = const.$

4. **inverters**: DC or variable AC sources with power electronics

   (i) have constant/controllable PQ

   (ii) or mimic generators with $M = 0$

Common variations in dynamic network models

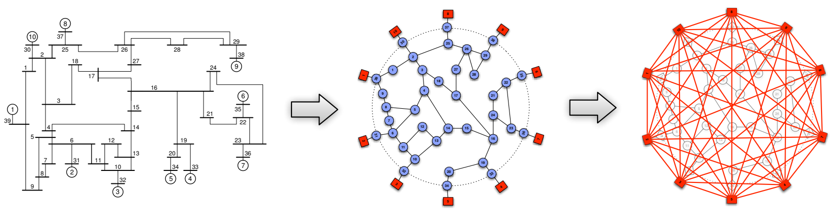
dynamic behavior is very much dependent on load models & generator models

1. frequency/voltage-depend. loads [A. Bergen & D. Hill '81, I. Hiskens & D. Hill '89, R. Davy & I. Hiskens '97]

$$D_i\dot{\theta}_i + P_i = -P_{i,\text{inj}}$$
$$f_i(\dot{V}_i) + Q_i = -Q_{i,\text{inj}}$$

2. network-reduced models after Kron reduction of loads [H. Chiang, F. Wu, & P. Varaiya '94] (very common but poor assumption: $G_{ij} = 0$)

$$M_i\ddot{\theta}_i + D\dot{\theta}_i = P_{i,\text{mech}} \underbrace{- \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j) - \sum\nolimits_j G_{ij}E_iE_j\cos(\theta_i - \theta_j)}_{\text{effect of resistive loads}}$$



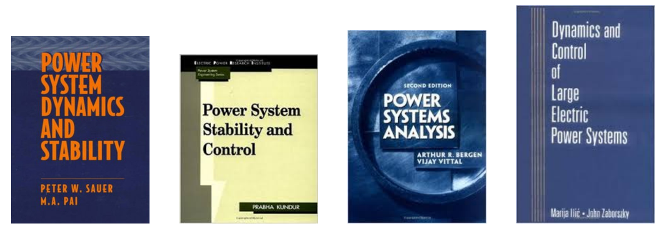
Common variations in dynamic network models — cont'd

dynamic behavior is very much dependent on load models & generator models

3. higher order generator dynamics [P. Sauer & M. Pai '98] — voltages, controls, magnetics etc. (reduction via singular perturbations)

4. dynamic & detailed load models [D. Karlsson & D. Hill '94] — aggregated dynamic load behavior (e.g., load recovery after voltage step)

5. time-domain models [S. Caliskan & P. Tabuada '14, S. Fiaz et al. '12] — passive Port-Hamiltonian models for machines & RLC circuitry



"Power system research is all about the art of making the right assumptions."

Outline

Decoupled Active Power Flow (Synchronization)

Synchronization & feasibility of active power flow

basic problem setup

- **structure-preserving power network model** [A. Bergen & D. Hill '81]:
  (simple dynamics & decoupled lossless flows capture essential phenomena)

synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

frequency-dependent loads: $D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

- **synchronization** = sync'd frequencies & bounded active power flows

$$\dot{\theta}_i = \omega_{\text{sync}}\ \forall\, i \in \mathcal{V} \quad \& \quad |\theta_i - \theta_j| \leq \gamma < \pi/2\ \forall\, \{i,j\} \in \mathcal{E}$$

= active power flow feasibility & security constraints

- **sync is crucial** for the functionality and operation of the power grid
- **explicit sync frequency:** if sync, then $\omega_{\text{sync}} = \sum_i P_i / \sum_i D_i$
  (by summing over all equations)

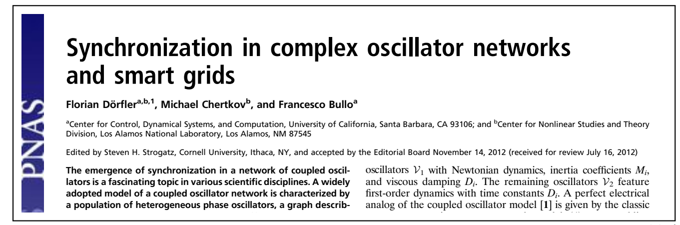
Synchronization & feasibility of active power flow

some key questions

Given: network parameters & topology and load & generation profile
Q: "∃ an optimal, stable, and robust sync'd operating point ?"

1. Security analysis [Araposthatis et al. '81, Wu et al. '80 & '82, Ilić '92, ...]
2. Load flow feasibility [Chiang et al. '90, Dobson '92, Lesieutre et al. '99, ...]
3. Optimal generation dispatch [Lavaei et al. '12, Bose et al. '12, ...]
4. Transient stability [Sastry et al. '80, Bergen et al. '81, Hill et al. '86, ...]
5. Inverters in microgrids [Chandorkar et. al. '93, Guerrero et al. '09, Zhong '11, ...]
6. Complex networks [Hill et al. '06, Strogatz '01, Arenas et al '08, ...]

Further reading on sync problem: (my perspective)


PNAS

Synchronization in complex oscillator networks and smart grids

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The emergence of synchronization in a network of coupled oscillators is a fascinating topic in various scientific disciplines. A widely adopted model of a coupled oscillator network is characterized by a population of heterogeneous phase oscillators, a graph describ-

oscillators $\mathcal{V}_1$ with Newtonian dynamics, inertia coefficients M_i, and viscous damping D_i. The remaining oscillators $\mathcal{V}_2$ feature first-order dynamics with time constants D_i. A perfect electrical analog of the coupled oscillator model [1] is given by the classic

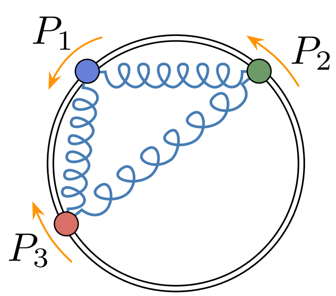
A perspective from coupled oscillators

Mechanical oscillator network

Angles $(\theta_1, \ldots, \theta_n)$ evolve on $\mathbb{T}^n$ as

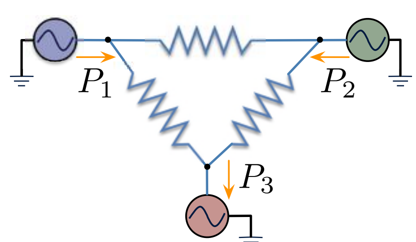
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$

- inertia constants $M_i > 0$
- viscous damping $D_i > 0$
- external torques $P_i \in \mathbb{R}$
- spring constants $B_{ij} \geq 0$



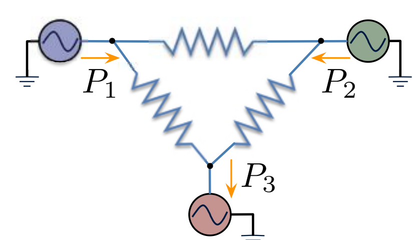
Structure-preserving power network

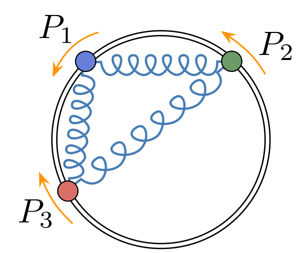
$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$
$$D_i\dot{\theta}_i = P_i - \sum_j B_{ij}\sin(\theta_i - \theta_j)$$



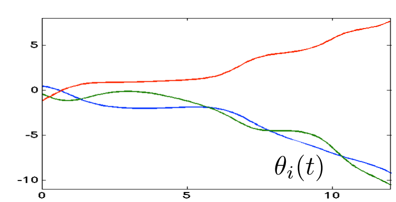
Phenomenology of sync in power networks

- sync is **crucial for AC power grids**

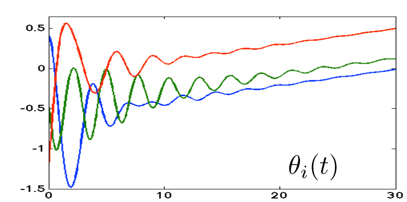




- sync is a **trade-off**



weak coupling & heterogeneous



strong coupling & homogeneous

Phenomenology of sync in power networks

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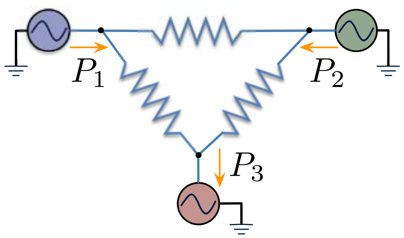

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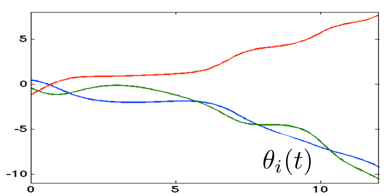
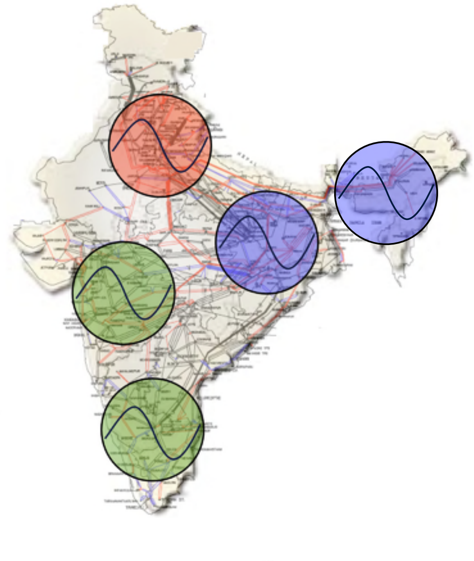


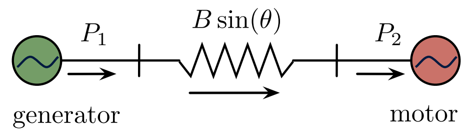
weak coupling & heterogeneous



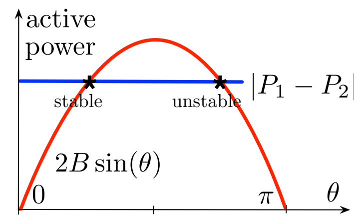
Blackout India July 30/31 2012

Back of the envelope calculations for the two-node case

generator connected to identical motor shows bifurcation at difference angle $\theta = \pi/2$



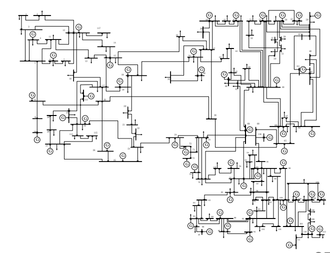
$$M\ddot{\theta} + D\dot{\theta} = P_1 - P_2 - 2B\sin(\theta)$$



$\exists$ stable sync $\Leftrightarrow$ $B > |P_1 - P_2|/2$ $\Leftrightarrow$ "ntwk coupling > heterogeneity"

Q1: Quantitative generalization to a complex & large-scale network?

Q2: What are the particular metrics for coupling and heterogeneity?



Primer on algebraic graph theory

for a connected and undirected graph

Laplacian matrix $L =$ "degree matrix" $-$ "adjacency matrix"

$$L = L^T = \begin{bmatrix} \vdots & \ddots & \vdots & \iddots & \vdots \\ -B_{i1} & \cdots & \sum_{j=1}^{n} B_{ij} & \cdots & -B_{in} \\ \vdots & \iddots & \vdots & \ddots & \vdots \end{bmatrix} \geq 0$$

is positive semidefinite with one zero eigenvalue & eigenvector $\mathbb{1}_n$

Notions of connectivity

- spectral: 2nd smallest eigenvalue of L is "algebraic connectivity" $\lambda_2(L)$
- topological: degree $\sum_{j=1}^{n} B_{ij}$ or degree distribution

Notions of heterogeneity

$\|P\|_{\mathcal{E},\infty} = \max_{\{i,j\}\in\mathcal{E}} |P_i - P_j|$, $\qquad \|P\|_{\mathcal{E},2} = \left(\sum_{\{i,j\}\in\mathcal{E}} |P_i - P_j|^2\right)^{1/2}$

Synchronization in "complex" networks

for a first-order model — all results generalize locally

$$\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$

1. **local stability** for equilibria satisfying $\quad |\theta_i^* - \theta_j^*| < \pi/2 \;\forall\, \{i,j\} \in \mathcal{E}$
   (linearization is Laplacian matrix)
2. **necessary sync condition**: $\quad \sum_j B_{ij} \geq |P_i - \omega_{\text{sync}}| \Leftarrow$ sync
   (so that syn'd solution exists)
3. **sufficient sync condition**: $\quad \lambda_2(L) > \|P\|_{\mathcal{E},2} \;\Rightarrow\;$ sync
   [FD & F. Bullo '12]

$\Rightarrow$ $\exists$ similar conditions with diff. metrics on coupling & heterogeneity

$\Rightarrow$ **Problem:** sharpest general conditions are conservative

A nearly exact sync condition [FD, M. Chertkov, & F. Bullo '13]

1. search equilibrium θ^* with $|\theta_i^* - \theta_j^*| \leq \gamma < \pi/2$ for all $\{i,j\} \in \mathcal{E}$:

$$P_i = \sum\nolimits_j B_{ij} \sin(\theta_i - \theta_j) \qquad (\star)$$

2. consider linear "small-angle" DC approximation of $(\star)$:

$$P_i = \sum\nolimits_j B_{ij}(\delta_i - \delta_j) \quad \Leftrightarrow \quad P = L\delta \qquad (\star\star)$$

unique solution (modulo symmetry) of $(\star\star)$ is $\delta^* = L^\dagger P$

3. solution ansatz for $(\star)$: $\boxed{\theta_i^* - \theta_j^* = \arcsin(\delta_i^* - \delta_j^*)}$ **(for a tree)**

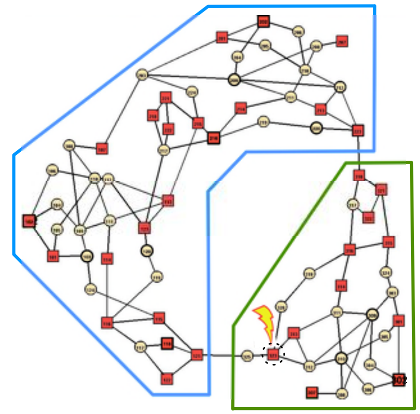
$$P_i = \sum\nolimits_{j=1}^n a_{ij} \sin(\theta_i - \theta_j) = \sum\nolimits_{j=1}^n a_{ij} \sin\big(\arcsin(\delta_i^* - \delta_j^*)\big) = P_i \quad \checkmark$$

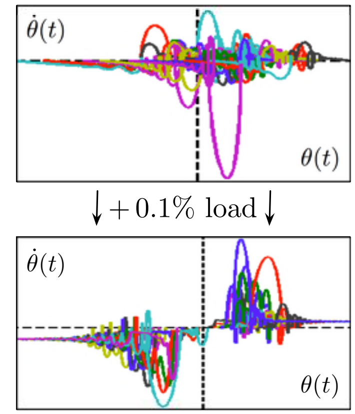
$\Rightarrow$ **Thm:** $\exists\, \theta^*$ with $|\theta_i^* - \theta_j^*| \leq \gamma \;\forall \{i,j\} \in \mathcal{E} \quad \Leftrightarrow \quad \|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$



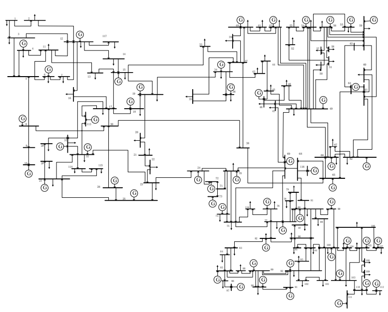


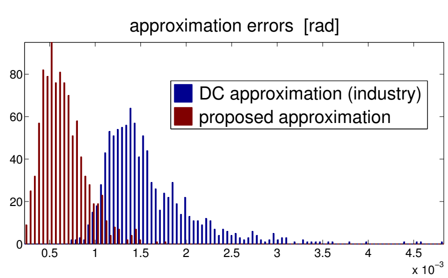
Reliability Test System RTS 96 under two loading conditions

Synchronization tests & power flow approximations

Sync cond': (heterogeneity)/(ntwk coupling) < (transfer capacity)

$\|L^\dagger P\|_{\mathcal{E},\infty} \leq \sin(\gamma)$ & new DC approx. $\theta \approx \arcsin(L^\dagger P)$





IEEE 118 bus system (Midwest)

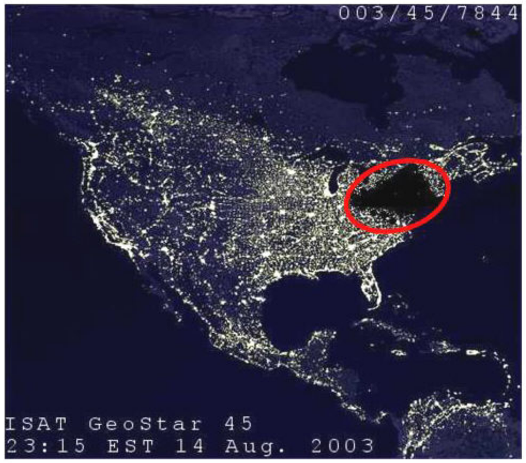
Outperforms conventional DC approximation "on average & in the tail".

Decoupled Reactive Power Flow (Voltage Collapse)

Voltage collapse in power networks

- **voltage instability**: loading > capacity ⇒ voltages drop
  "mainly" a reactive power phenomena
- **recent outages:** Québec '96, Scandinavia '03, Northeast '03, Athens '04

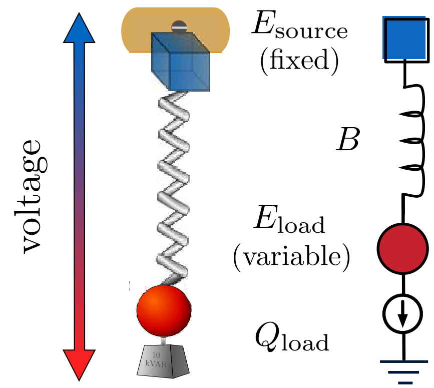
> *"Voltage collapse is still the biggest single threat to the transmission system. It's what keeps me awake at night."*
> – Phil Harris, CEO PJM.

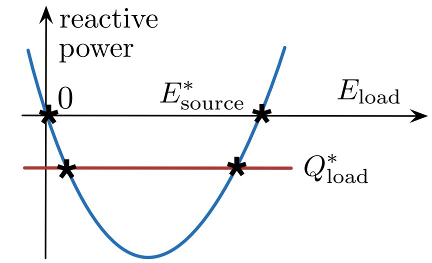


Back of the envelope calculations for the two-node case

source connected to load shows bifurcation at load voltage $E_{\text{load}} = E_{\text{source}}/2$

reactive power balance at load: $Q_{\text{load}} = B\, E_{\text{load}}(E_{\text{load}} - E_{\text{source}})$



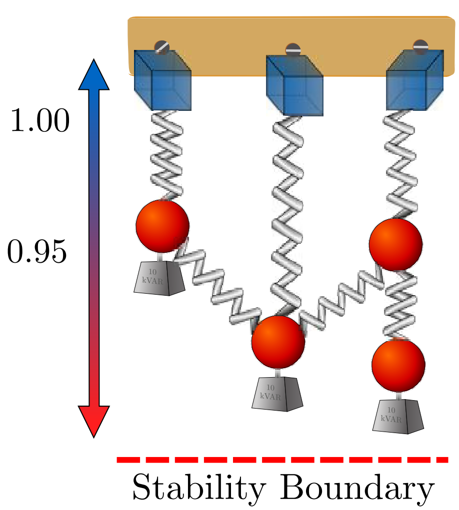


$E_{\text{load}} \in \mathbb{R} \Leftrightarrow Q_{\text{load}} \geq -B\,(E_{\text{source}})^2/4$

∃ high load voltage solution ⇔ (load) < (network)(source voltage)$^2/4$

Intuition extends to complex networks – essential insights

Reactive power balance:
$Q_i = -\sum_j B_{ij} E_i E_j$



Suff. & tight cond' for general case [J. Simpson-Porco, FD, & F. Bullo, '14]:

∃ unique high-voltage solution E_{load}

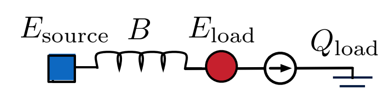
⇔

$$\frac{4 \cdot \text{load}}{(\text{admittance})(\text{nominal voltage})^2} < 1$$

1. nominal (zero load) voltage E_{nom}
   $$0 = -\sum_j B_{ij}\, E_{i,\text{nom}}\, E_{j,\text{nom}}$$
2. coord-trafo to solution guess:
   $$x_i = E_i / E_{i,\text{nom}} - 1$$
3. Picard-Banach iteration $x^+ = f(x)$

More back of the envelope calculations

$Q_{\text{load}} = B\, E_{\text{load}}(E_{\text{load}} - E_{\text{source}})$



∃ closed-form sol': $E_{\text{load}} = E_{\text{source}}\left(1/2 \pm 1/2\sqrt{1 + 4Q_{\text{load}}/(BE^2_{\text{source}})}\right)$

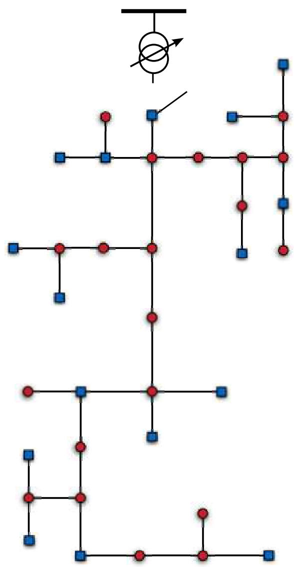
⇒ Taylor exp. for $E_{\text{source}} \to \infty$ (or $Q_{\text{load}} \to 0$): $E_{\text{load}} \approx E_{\text{source}} + \frac{Q_{\text{load}}}{BE_{\text{source}}}$

- **General case**: existence & approximation from implicit function thm
  - if all loads Q_i are "sufficiently small" [D. Molzahn, B. Lesieutre, & C. DeMarco '12]
  - if slack bus has "sufficiently large" E_{source} [S. Bolognani & S. Zampieri '12 & '14]
  - if each source is above a "sufficiently large" E_{source} [B. Gentile et al. '14]
  - if previous existence condition is met [J. Simpson-Porco, FD, & F. Bullo, '14]

⇒ 1st order approximation: $E_{\text{load}} \approx E_{\text{source}}\mathbb{1} + \frac{1}{E_{\text{source}}} B^{-1} Q_{\text{load}}$

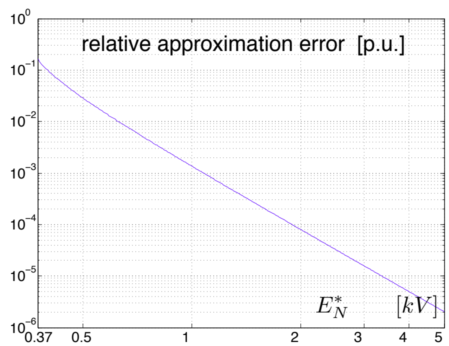
Linear DC approximation extends to complex networks

verification via *IEEE 37 bus distribution system* (SoCal)

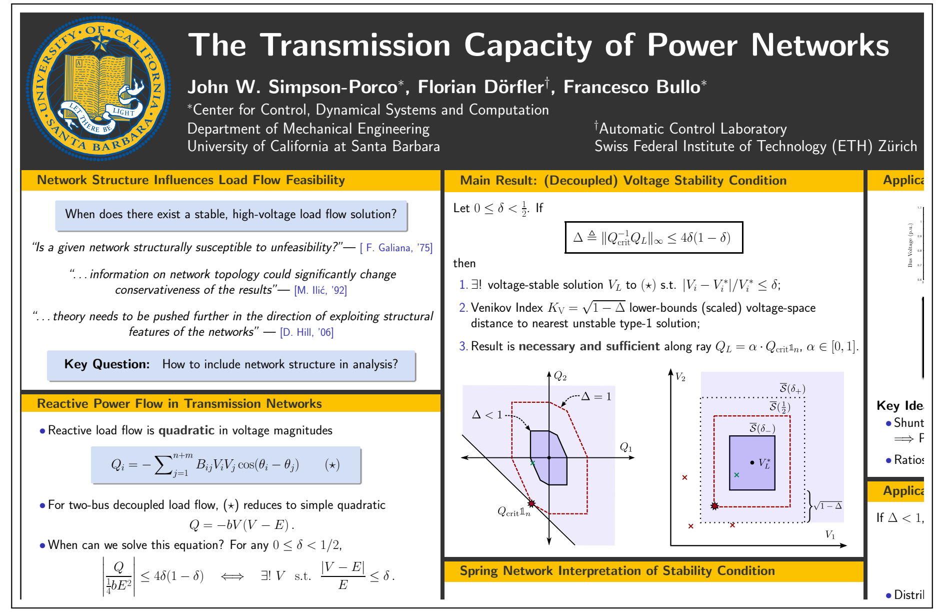


DC approximation [Gentile, Simpson-Porco, Dörfler, Zampieri, & Bullo, '14]:

$$E_{\text{load}} \approx E_{\text{source}}\mathbb{1} + B^{-1}Q_{\text{load}}/E_{\text{source}} + \mathcal{O}\left(1/E_{\text{source}}^{*}{}^{3}\right)$$



More on reactive power, voltage collapse & approximations


The Transmission Capacity of Power Networks

John W. Simpson-Porco*, Florian Dörfler†, Francesco Bullo*

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Network Structure Influences Load Flow Feasibility

When does there exist a stable, high-voltage load flow solution?

"Is a given network structurally susceptible to unfeasibility?" — [F. Galiana, '75]

"... information on network topology could significantly change conservativeness of the results" — [M. Ilić, '92]

"... theory needs to be pushed further in the direction of exploiting structural features of the networks" — [D. Hill, '06]

Key Question: How to include network structure in analysis?

Reactive Power Flow in Transmission Networks

- Reactive load flow is **quadratic** in voltage magnitudes

$$Q_i = -\sum_{j=1}^{n+m} B_{ij}V_iV_j\cos(\theta_i-\theta_j) \qquad (\star)$$

- For two-bus decoupled load flow, $(\star)$ reduces to simple quadratic $Q = -bV(V-E)$.
- When can we solve this equation? For any $0 \leq \delta < 1/2$,

$$\left|\frac{Q}{\frac{1}{4}bE^2}\right| \leq 4\delta(1-\delta) \iff \exists!\, V \text{ s.t. } \frac{|V-E|}{E} \leq \delta.$$

Main Result: (Decoupled) Voltage Stability Condition

Let $0 \leq \delta < \frac{1}{2}$. If

$$\Delta \triangleq \|Q_{\text{crit}}^{-1}Q_L\|_\infty \leq 4\delta(1-\delta)$$

then

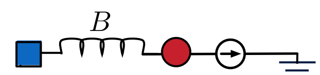
1. $\exists!$ voltage-stable solution V_L to $(\star)$ s.t. $|V_i - V_i^*|/V_i^* \leq \delta$;
2. Venikov Index $K_V = \sqrt{1-\Delta}$ lower-bounds (scaled) voltage-space distance to nearest unstable type-1 solution;
3. Result is **necessary and sufficient** along ray $Q_L = \alpha \cdot Q_{\text{crit}}\mathbb{1}_n$, $\alpha \in [0,1]$.

Spring Network Interpretation of Stability Condition

Coupled & Lossy Power Flow

Simplest example shows surprisingly complex behavior

- PV source, PQ load, & lossless line

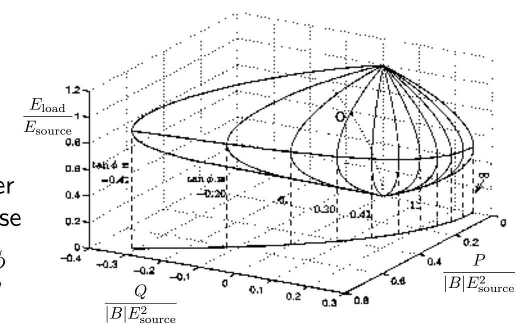


$$P = B\,E_{\text{source}}\,E_{\text{load}}\sin(\theta)$$
$$Q = B\,E_{\text{load}}^2 - B\,E_{\text{source}}\,E_{\text{load}}\cos(\theta)$$

- after eliminating θ, there exists $E_{\text{load}} \in \mathbb{R}_{\geq 0}$ if and only if

$$P^2 - B\,E_{\text{source}}^2\,Q \leq B^2E_{\text{source}}^4/4$$

- Observations:
  1. $P = 0$ case consistent with previous decoupled analysis
  2. $Q = 0$ case delivers 1/2 transfer capacity from decoupled case
  3. intermediate cases $Q = P\tan\phi$ give so-called "nose curves"



Coupled & lossy power flow in complex networks

- active power: $P_i = \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i = -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

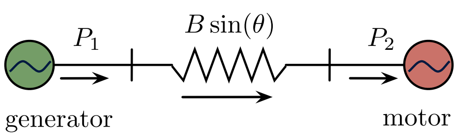
- what makes it so much harder than the previous two node case?
  losses, mixed lines, cycles, PQ-PQ connections, ...
- much theoretic work, qualitative understanding, & numeric approaches:
  - existence of solutions [Thorp, Schulz, & Ilić '86, Wu & Kumagai '82]
  - solution space [Hiskens & Davy '01, Overbye & Klump '96, Van Cutsem '98, ...]
  - distance-to-failure [Venikov '75, Abe & Isono '76, Dobson '89, Andersson & Hill '93, ...]
  - convex relaxation approaches [Molzahn, Lesieutre, & DeMarco '12]
- little analytic & quantitative understanding beyond the two-node case

"Whoever figures that one out wins a noble prize!" Pete Sauer

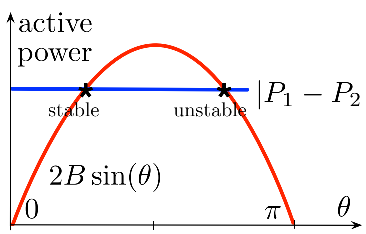
Transient Rotor Angle Stability

Revisit of the two-node case — the forced pendulum

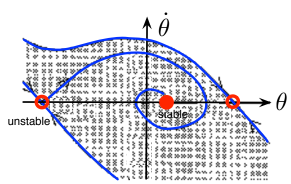
more complex than anticipated



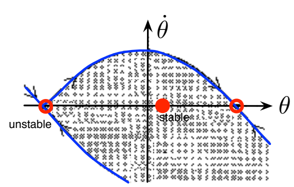
$$\dot\theta = \omega$$
$$M\dot\omega = -D\omega + P_1 - P_2 - 2B\sin(\theta)$$



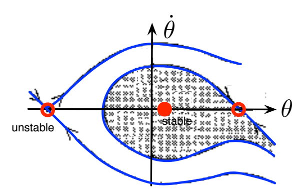
- **Local stability**: $\exists$ local stable solution $\Leftrightarrow$ $B > |P_1 - P_2|/2$
- **Global stability**: depends on gap $B > |P_1 - P_2|/2$ and D/M ratio



$(D/M) > (D/M)_{\text{critical}}$



$(D/M) = (D/M)_{\text{critical}}$

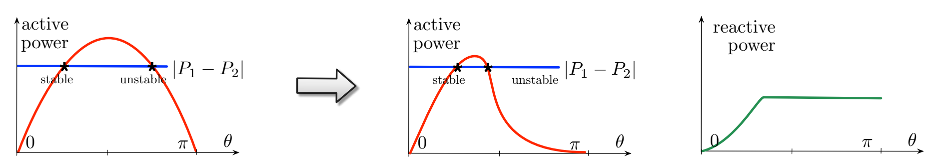


$(D/M) < (D/M)_{\text{critical}}$

Revisit of the two-node case — cont'd

the story is not complete ... some further effects that we swept under the carpet

- **Voltage reduction**: to maintain a constant voltage, a generator needs to provide reactive power. When encountering the maximum reactive power support, the generator becomes a PQ bus and voltage drops.



- **Load sensitivity**: different behavior depending on load model: resistive, constant power, frequency-dependent, dynamic, power electronics, ...
- **Singularity-issues** for coupled power flows (load voltage collapse)
- **Losses & higher-order dynamics** change stability properties ...

$\Rightarrow$ quickly run into computational approaches

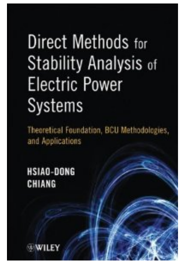
Transient stability in multi-machine power systems

generators:

$$\dot{\theta}_i = \omega_i$$
$$M_i\dot{\omega}_i = -D_i\omega_i + P_i - \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$

loads:

$$D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}E_iE_j\sin(\theta_i - \theta_j)$$
$$Q_i = -\sum\nolimits_j B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$



Challenge (improbable): faster-than-real-time transient stability assessment

Energy function methods for simple lossless models via Lyapunov function

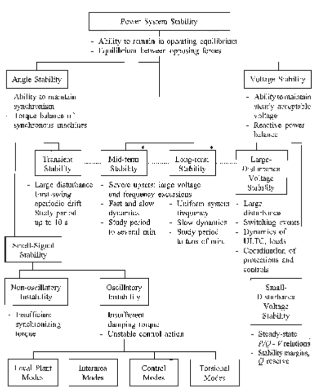
$$V(\omega,\theta,E) = \sum\nolimits_i \frac{1}{2}M_i\omega_i^2 - \sum\nolimits_i P_i\theta_i - \sum\nolimits_i Q_i\log E_i - \sum\nolimits_{ij} B_{ij}E_iE_j\cos(\theta_i - \theta_j)$$

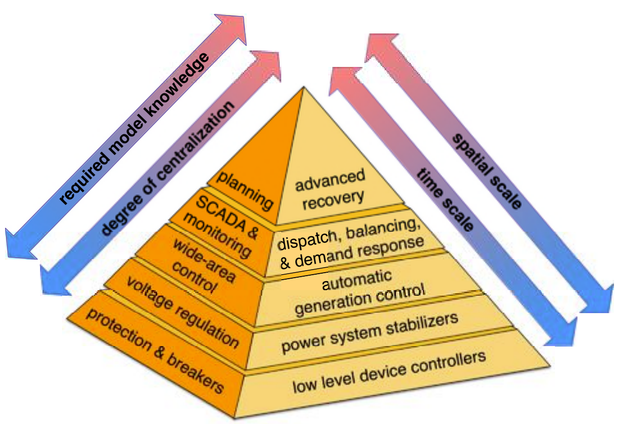
Computational approaches: level sets of energy functions & unstable equilibria, sum-of-squares methods, convex optimization approaches, time-domain simulations, ... (more later this week)

Outline

A plethora of control tasks and nested control layers

organized in hierarchy and separated by states & spatial/temporal/centralization scales



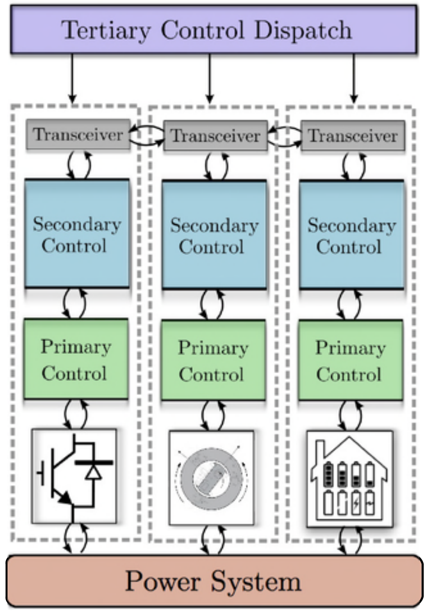


We will focus on frequency control & primary/secondary/tertiary layers.

All dynamics & controllers are interacting. Classification & hierarchy are for simplicity.

Objectives

Hierarchical frequency control architecture & objectives



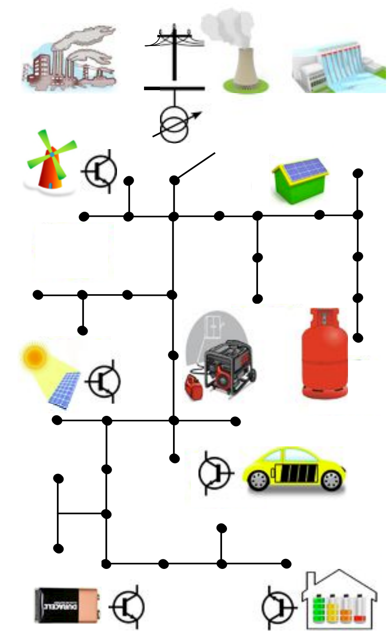
3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast
2. **Secondary control** (minutes)
   - Goal: maintain operating point in presence of disturbances
   - Strategy: centralized
1. **Primary control** (real-time)
   - Goal: stabilize frequency & share unknown load
   - Strategy: decentralized

Q: Is this layered & hierarchical architecture still appropriate for tomorrow's power system?

Is the hierarchical control architecture still appropriate?

Some recent developments
- increasing renewable integration
- synchronous machines replaced by power electronics sources
- bulk generation replaced by distributed low-inertia sources
- deregulated energy markets
- low gas prices & substitutions

Some "new" scenarios
- alternative spinning reserves: storage, load control, & DER
- networks of low-inertia & distributed renewable sources
- small-footprint islanded systems

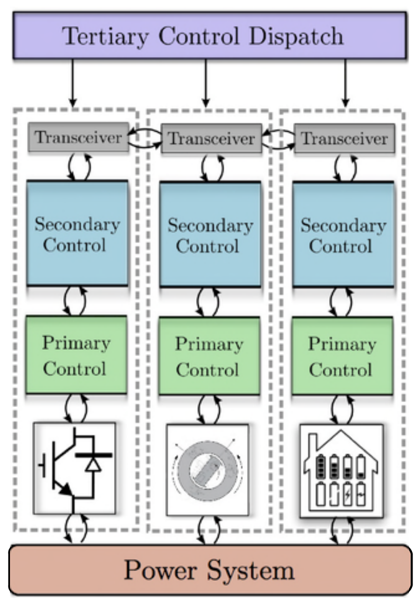


Need to adapt the control hierarchy in tomorrow's grid

☹perational challenges
- more uncertainty & less inertia
- more volatile & faster fluctuations
- plug'n'play control: fast, model-free, & without central authority

☺pportunities
- re-instrumentation: comm & sensors
- more & faster spinning reserves
- advances in control of cyber-physical & complex systems

⇒ **break** vertical & horizontal **hierarchy**

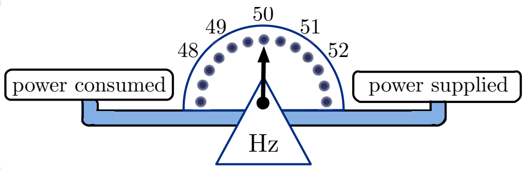


Primary Control

Decentralized primary control of active power

Emulate physics of dissipative coupled **synchronous machines**:

$$M_i\ddot{\theta} + D_i\dot{\theta}_i = P_i^* - \sum\nolimits_j B_{ij}\sin(\theta_i - \theta_j)$$

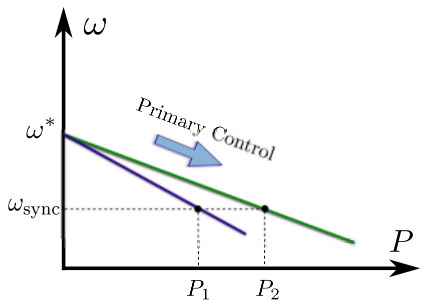


Conventional wisdom: physics are naturally stable & sync frequency reveals power imbalance

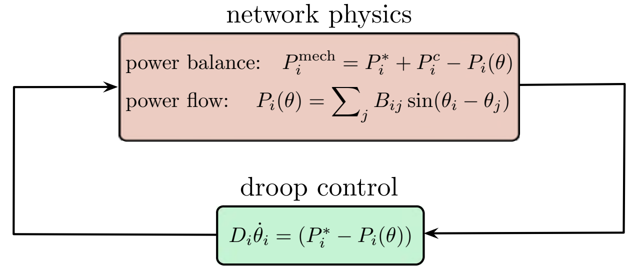
recall: $\omega_{\text{sync}} = \sum_i P_i^* / D_i$

$P/\dot{\theta}$ droop control:

$$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$$
$$\Updownarrow$$
$$D_i\dot{\theta}_i = P_i^* - P_i(\theta)$$



Putting the pieces together...



synchronous machines: $M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

inverter sources & controllable loads: $D_i\dot{\theta}_i = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

passive loads & power-point tracking sources: $0 = P_i^* - \sum_j B_{ij}\sin(\theta_i - \theta_j)$

Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

$\exists$ unique & exp. stable frequency sync $\iff$ active power flow is feasible

Main **proof ideas** and some **further results**:

- stability via Jacobian arguments (as before)
- synchronization frequency: ($\propto$ power balance)
  $$\omega_{\text{sync}} = \omega^* + \frac{\sum_{\text{sources}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{sources}} D_i}$$
- steady-state power injections: (depend on D_i & P_i^*)
  $$\mathcal{P}_i = \begin{cases} P_i^* & \text{(load } \#i) \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & \text{(source } \#i) \end{cases}$$

power sharing & economic optimality under droop control

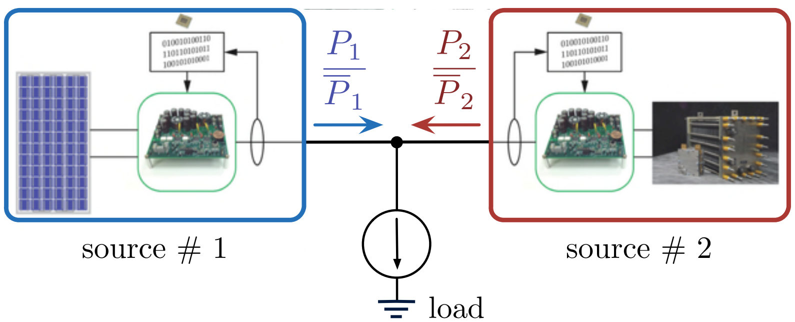
(sometimes in tertiary layer)

Objective I: decentralized proportional load sharing

1) Sources have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \le \left| \sum_{\text{loads}} P_j^* \right| \le \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$



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3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

A little calculation reveals in steady state:

$$\frac{P_i(\theta)}{\overline{P}_i} \overset{!}{=} \frac{P_j(\theta)}{\overline{P}_j} \quad \Rightarrow \quad \frac{P_i^* - (D_i \omega_{\text{sync}} - \omega^*)}{\overline{P}_i} \overset{!}{=} \frac{P_j^* - (D_j \omega_{\text{sync}} - \omega^*)}{\overline{P}_i}$$

... so choose

$$\frac{P_i^*}{\overline{P}_i} = \frac{P_j^*}{\overline{P}_j} \quad \text{and} \quad \frac{D_i}{\overline{P}_i} = \frac{D_j}{\overline{P}_j}$$

Objective I: decentralized proportional load sharing

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2) Load must be **serviceable**: $0 \le \left| \sum_{\text{loads}} P_j^* \right| \le \sum_{\text{sources}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$\boxed{D_i / \overline{P}_i = D_j / \overline{P}_j \;\; \& \;\; P_i^* / \overline{P}_i = P_j^* / \overline{P}_j}$$

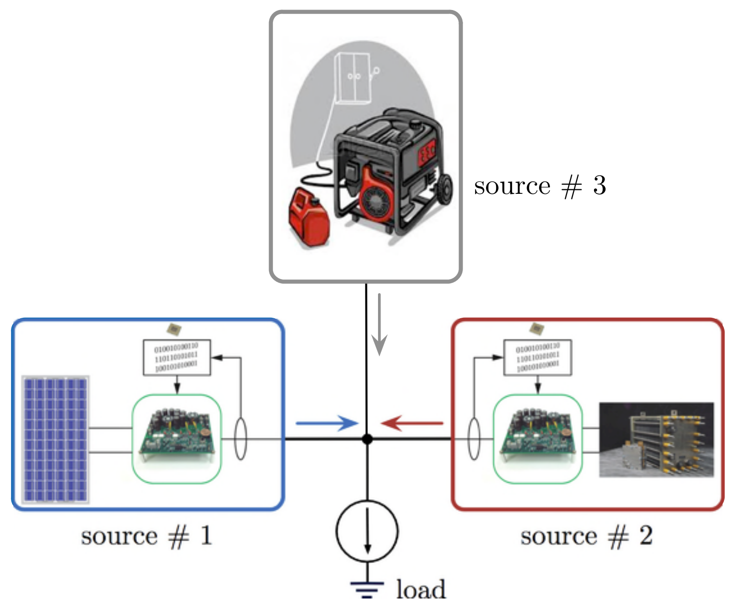
The the following statements hold:

(i) Proportional load sharing: $P_i(\theta) \,/\, \overline{P}_i = P_j(\theta) \,/\, \overline{P}_j$

(ii) Constraints met: $0 \le \left| \sum_{\text{loads}} P_j^* \right| \le \sum_{\text{sources}} \overline{P}_j \;\Leftrightarrow\; P_i(\theta) \in [0, \overline{P}_i]$

Objective I: fair proportional load sharing

proportional load sharing is not always the right objective



Objective II: optimal power flow = tertiary control

an offline resource allocation/scheduling problem

minimize	{cost of generation, losses, ...}
subject to	
equality constraints:	power balance equations
inequality constraints:	flow/injection/voltage constraints
logic constraints:	commit generators yes/no
	$\vdots$

Will be discussed in detail later.

Objective II: simple economic dispatch

minimize the total accumulated generation (many variations possible)

$$\text{minimize}_{\theta\in\mathbb{T}^n,\,u\in\mathbb{R}^{n_I}} \quad f(u) = \sum\nolimits_{\text{sources}} \alpha_i u_i^2$$

subject to

source power balance:	$P_i^* + u_i = P_i(\theta)$
load power balance:	$P_i^* = P_i(\theta)$
branch flow constraints:	$\lvert\theta_i - \theta_j\rvert \le \gamma_{ij} < \pi/2$

Unconstrained case: identical marginal costs $\alpha_i u_i^* = \alpha_j u_j^*$ at optimality

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In a grid with distributed energy resources, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

Objective II: decentralized dispatch optimization

Theorem: optimal droop [FD, Simpson-Porco, & Bullo '13, Zhao, Mallada, & FD '14]

The following statements are equivalent:

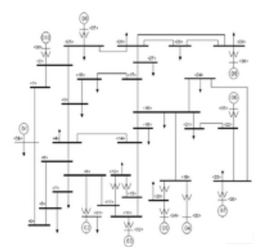
(i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*).

(ii) $\exists$ droop coefficients D_i such that the power system possesses a unique & locally exp. stable sync'd solution θ.

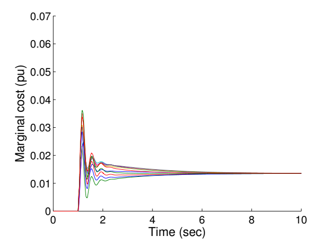
If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $\boxed{D_i\alpha_i = D_j\alpha_j}$.

- includes proportional load sharing $\alpha_i \propto 1/\overline{P}_i$
- similar results hold for strictly convex cost & general **constrained** case
- similar results in transmission ntwks with DC flow [E. Mallada & S. Low, '13] & [N. Li, L. Chen, C. Zhao, & S. Low '13] & [X. Zhang & A. Papachristodoulou, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & ...

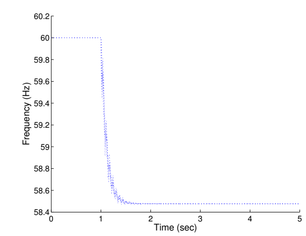
Some quick simulations & extensions



IEEE 39 New England with load step at 1s



$t \to \infty$: convergence to identical marginal costs

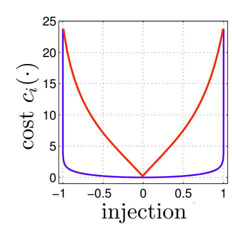


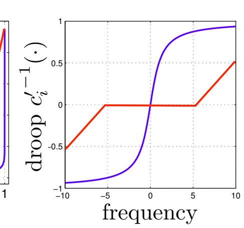
$t \to \infty$: frequency $\propto$ power imbalance

$\Rightarrow$ strictly convex & differentiable cost $f(u) = \sum_{\text{sources}} c_i(u_i)$

$\Rightarrow$ non-linear frequency droop curve $c_i'^{-1}(\dot{\theta}_i) = P_i^* - P_i(\theta)$

$\Rightarrow$ include dead-bands, saturation, etc.

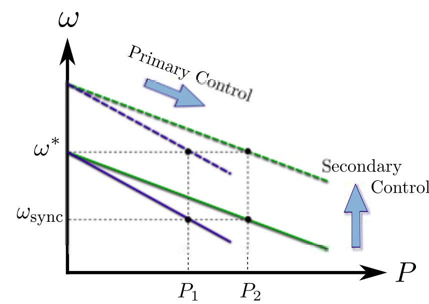




Secondary Control

Secondary frequency control

- **Problem:** steady-state frequency deviation ($\omega_{\text{sync}} \neq \omega^*$)
- **Solution:** integral control of frequency error
- **Basics** of integral control $\boxed{\frac{1}{s}}$:



1. discrete time: $\boxed{u_i(t+1) = u_i(t) + k \cdot \dot{\theta}_i(t)}$ with gain $k > 0$
2. continuous-time: $\boxed{u_i(t) = k \cdot \int_0^t \dot{\theta}_i(\tau)\, d\tau}$ or $\boxed{\dot{u}_i(t) = k \cdot \dot{\theta}_i(t)}$

$\Rightarrow$ $\dot{\theta}_i(t)$ is zero in (a possibly stable) steady state

$\Rightarrow$ add additional injection $u_i(t)$ to droop control

Decentralized secondary integral frequency control

$\boxed{\frac{1}{s}}$ add local integral controller to every droop controller

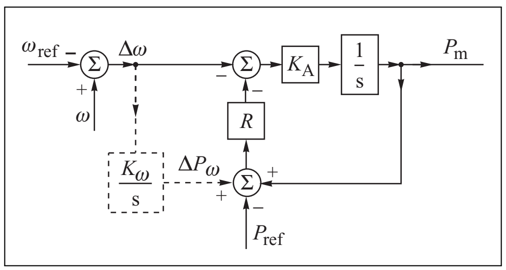
$\Rightarrow$ stable closed-loop & zero frequency deviation ✓

$\Rightarrow$ sometimes globally stabilizing [C. Zhao, E. Mallada, & FD, '14] ✓

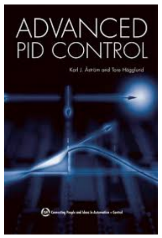
☹ every integrator induces a 1d equilibrium subspace

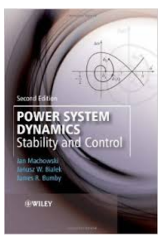
☹ injections live in subspace of dimension # integrators

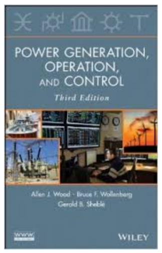
☹ load sharing & economic optimality are lost ...



turbine governor integral control loop

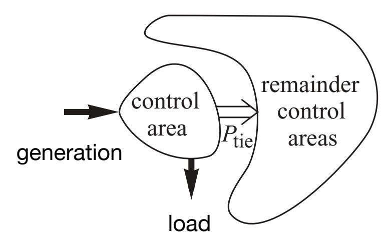






Automatic generation control (AGC)

- **ACE** area control error = { frequency error } + { generation - load - tie-line flow }



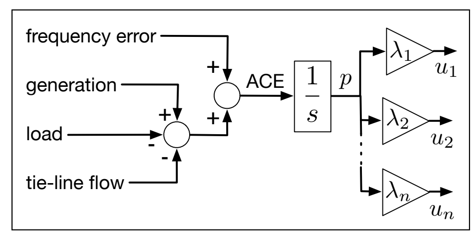
$\boxed{\frac{1}{s}}$ **centralized integral control:**

$$p(t) = \int_0^t \text{ACE}(\tau)\, d\tau$$

- **generation allocation:** $u_i(t) = \lambda_i p(t)$, where λ_i is generation participation factor (in our case $\lambda_i = 1/\alpha_i$)

$\Rightarrow$ assures identical marginal costs: $\alpha_i u_i = \alpha_j u_j$

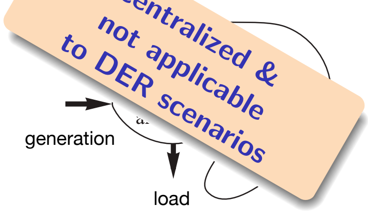
☺ load sharing & economic optimality are recovered



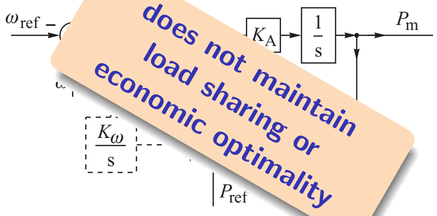
AGC implementation

Drawbacks of conventional secondary frequency control

Interconnected Systems	Isolated Systems
• c…tralized automatic …n control (AGC)	• dec…tralized PI control



centralized & not applicable to DER scenarios



does not maintain load sharing or economic optimality

Distributed energy ressources require **distributed** (**!**) secondary control.

An incomplete literature review of a busy field

ntwk with unknown disturbances ∪ integral control ∪ distributed averaging

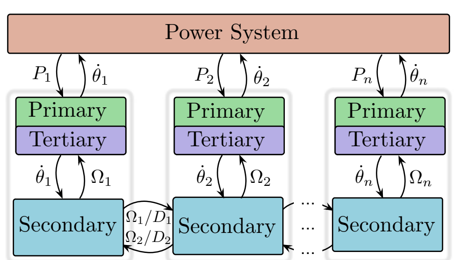
- all-to-all source frequency & injection averaging [Q. Shafiee, J. Vasquez, & J. Guerrero, '13] & [H. Liang, B. Choi, W. Zhuang, & X. Shen, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '12]
- optimality w.r.t. economic dispatch [E. Mallada & S. Low, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & [X. Zhang and A. Papachristodoulou, '13] & [N. Li, L. Chen, C. Zhao & S. Low '13]
- ratio consensus & dispatch [S.T. Cady, A. Garcıa-Domınguez, & C.N. Hadjicostis, '13]
- load balancing in Port-Hamiltonian networks [J. Wei & A. Van der Schaft, '13]
- passivity-based network cooperation and flow optimization [M. Bürger, D. Zelazo, & F. Allgöwer, '13, M. Bürger & C. de Persis '13, He Bai & S.Y. Shafi '13]
- distributed PI avg optimization [G. Droge, H. Kawashima, & M. Egerstedt, '13]
- PI avg consensus [R. Freeman, P. Yang, & K. Lynch '06] & [M. Zhu & S. Martinez '10]
- decentralized "practical" integral control [N. Ainsworth & S. Grijalva, '13]

The following idea precedes most references, it's simpler, & it's more robust.

Distributed **A**veraging **PI** (DAPI) control

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{sources}} a_{ij} \cdot (\alpha_i\Omega_i - \alpha_j\Omega_j)$$



- no tuning & no time-scale separation: $k_i, D_i > 0$
- distributed & modular: connected comm. ⊆ sources
- recovers primary op. cond. (load sharing & opt. dispatch)

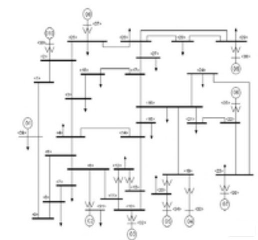
⇒ plug'n'play implementation

Theorem: stability of DAPI

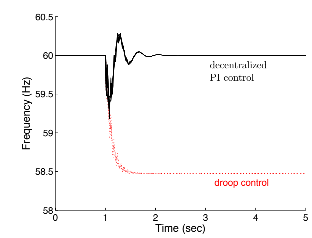
[J. Simpson-Porco, FD, & F. Bullo, '12]
[C. Zhao, E. Mallada, & FD '14]

primary droop controller works
⟺
secondary DAPI controller works

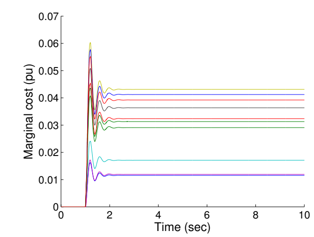
Simulations cont'd



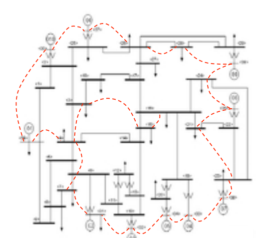
IEEE 39 New England with decentralized PI control



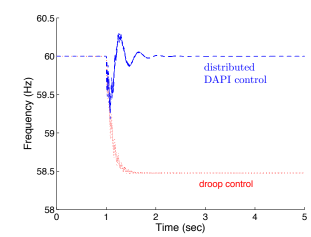
$t \to \infty$: decentralized PI control regulates frequency



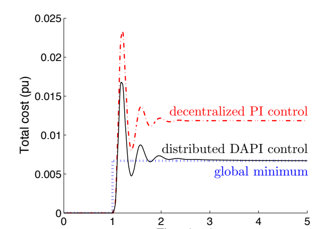
$t \to \infty$: decentralized PI control is not optimal



IEEE 39 New England with distributed DAPI control



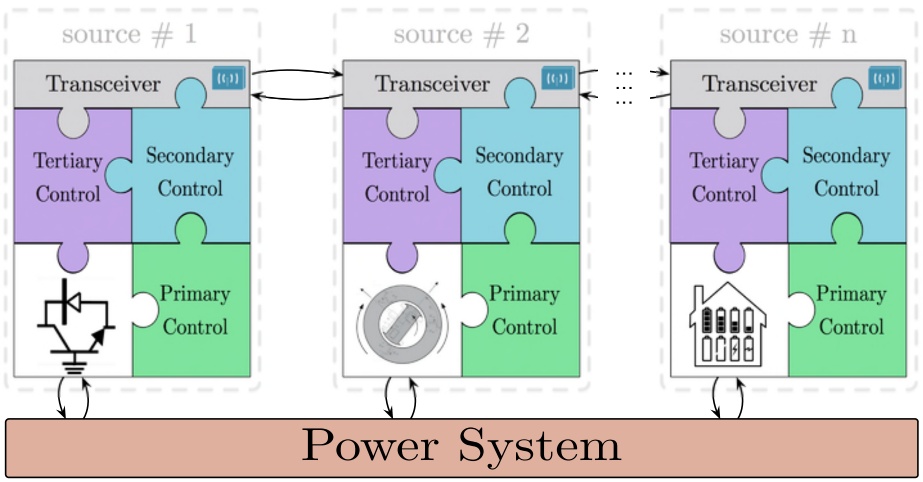
$t \to \infty$: DAPI control regulates frequency



DAPI control minimizes cost with little effort

Plug'n'play architecture

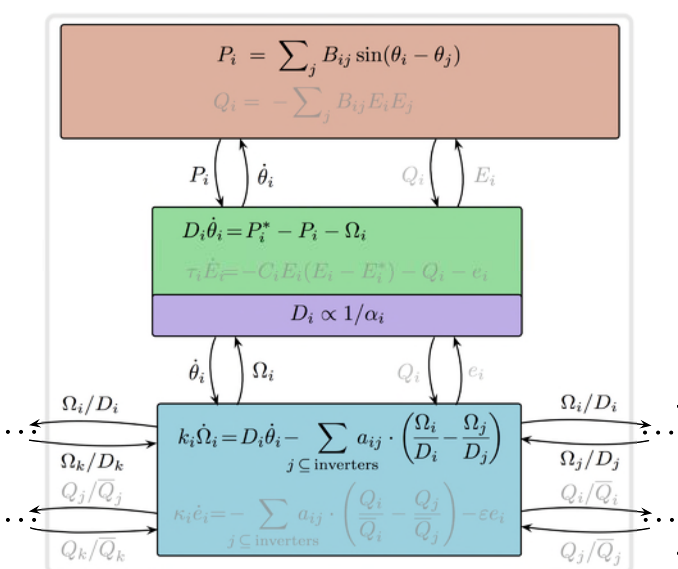
flat hierarchy, distributed, no time-scale separations, & model-free



plug-and-play experiments

Plug'n'play architecture

recap of detailed signal flow (active power only)



Power system:
physics
& loadflow

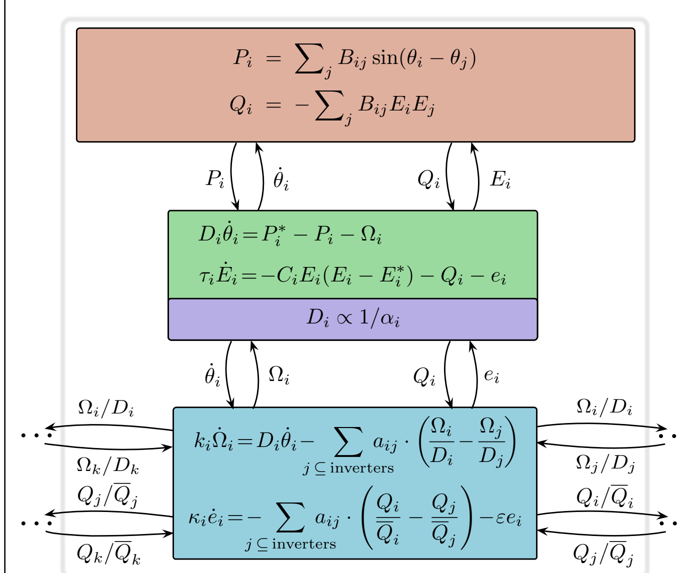
Primary control:
mimic oscillators
& polyn. symmetry

Tertiary control:
marginal costs
$\propto$ 1 /control gains

Secondary control:
diffusive averaging
of injection ratios

Plug'n'play architecture

recap of detailed signal flow (with reactive power)



Power system:
physics
& loadflow

Primary control:
mimic oscillators
& polyn. symmetry

Tertiary control:
marginal costs
$\propto$ 1 /control gains

Secondary control:
diffusive averaging
of injection ratios

Plug'n'play architecture

experiments also work well in the coupled & lossy case

$$P_i = \sum_j B_{ij}E_iE_j \sin(\theta_i - \theta_j) + G_{ij}E_iE_j \cos(\theta_i - \theta_j)$$
$$Q_i = -\sum_j B_{ij}E_iE_j \cos(\theta_i - \theta_j) + G_{ij}E_iE_j \sin(\theta_i - \theta_j)$$

P_i $\dot{\theta}_i$ Q_i E_i

$$D_i\dot{\theta}_i = P_i^* - P_i - \Omega_i$$
$$\tau_i\dot{E}_i = -C_iE_i(E_i - E_i^*) - Q_i - e_i$$
$$D_i \propto 1/\alpha_i$$

$\dot{\theta}_i$ Ω_i Q_i e_i

Ω_i/D_i Ω_k/D_k Ω_i/D_i Ω_j/D_j

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{inverters}} a_{ij} \cdot \left(\frac{\Omega_i}{D_i} - \frac{\Omega_j}{D_j}\right)$$

$Q_j/\overline{Q}_j$ $Q_k/\overline{Q}_k$ $Q_i/\overline{Q}_i$ $Q_j/\overline{Q}_j$

$$\kappa_i\dot{e}_i = -\sum_{j \subseteq \text{inverters}} a_{ij} \cdot \left(\frac{Q_i}{\overline{Q}_i} - \frac{Q_j}{\overline{Q}_j}\right) - \varepsilon e_i$$

Power system: physics & loadflow

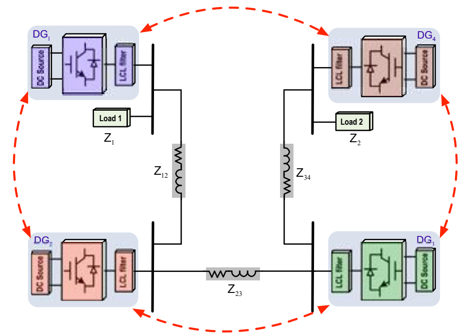
Primary control: mimic oscillators & polyn. symmetry

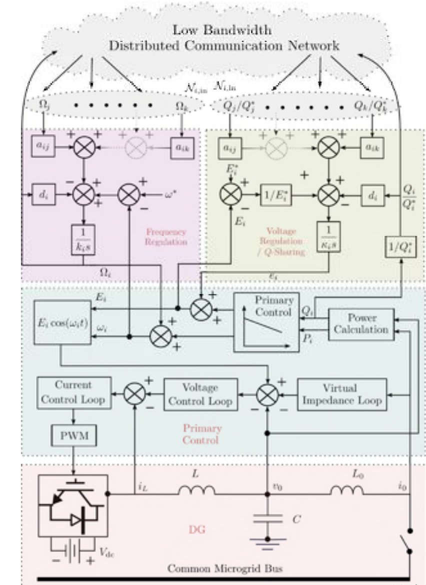
Tertiary control: marginal costs $\propto$ 1 /control gains

Secondary control: diffusive averaging of injection ratios

Experimental validation of control & opt. algorithms

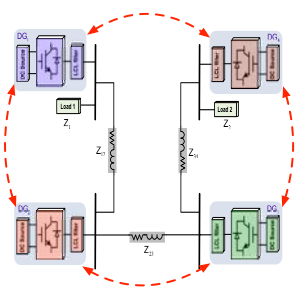
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University





Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing

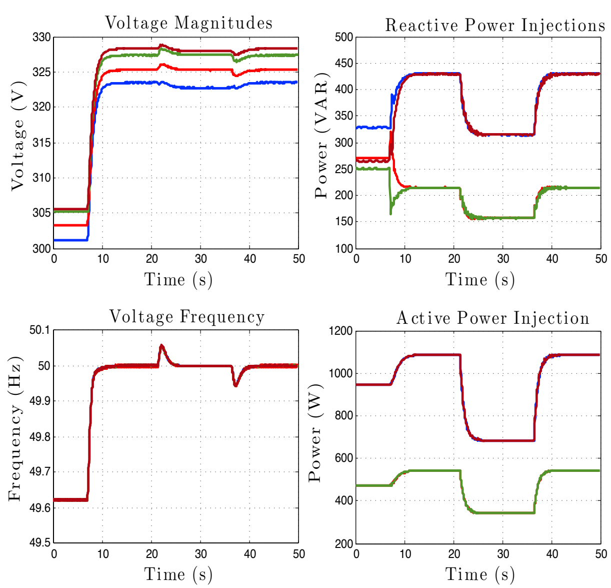


$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

$t = 22s$: load # 2 unplugged

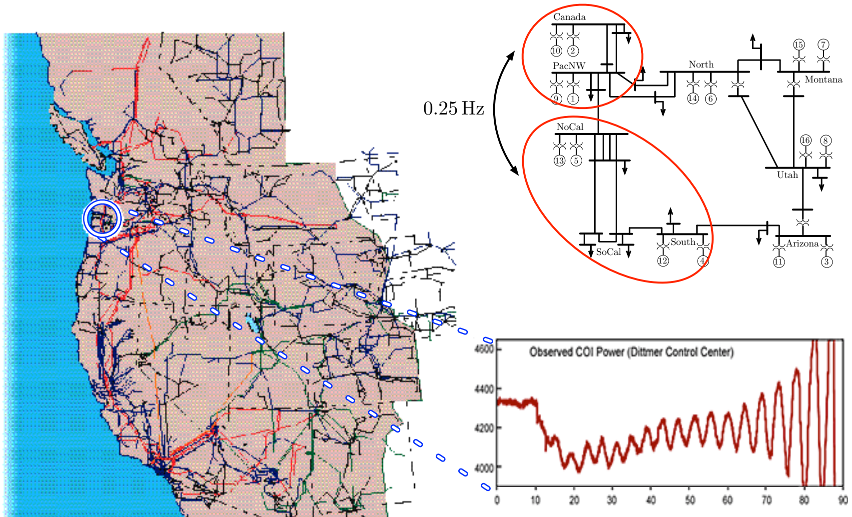
$t = 36s$: load # 2 plugged back



Outline

Electro-Mechanical Oscillations in Power Networks

- **Dramatic consequences:** blackout of August 10, 1996, resulted from instability of the 0.25 Hz mode in the Western interconnected system



Source: http://certs.lbl.gov

Causes for Oscillations

Power network swing dynamics

- Coarse-grained power network dynamics = generator **swing dynamics**:

$$M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_i - \sum\nolimits_j B_{ij}E_iE_j \sin(\theta_i - \theta_j)$$

- Swing equations **linearized** around an equilibrium $(\theta^*, \mathbf{0})$:

$$\boxed{M\ddot{\theta} + D\dot{\theta} + L\theta = 0}$$

M & $D \in \mathbb{R}^{n\times n}$ diagonal inertia and damping matrices

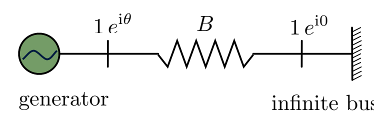
$L \in \mathbb{R}^{n\times n}$ Laplacian matrix with coupling $a_{ij} = E_i^* E_j^* B_{ij} \cos(\theta_i^* - \theta_j^*)$

$$L = \begin{bmatrix} \vdots & \ddots & \vdots & \cdot^{\cdot^{\cdot}} & \vdots \\ -a_{i1} & \cdots & \sum_{j=1}^{n} a_{ij} & \cdots & -a_{in} \\ \vdots & \cdot^{\cdot^{\cdot}} & \vdots & \ddots & \vdots \end{bmatrix}$$

$\Rightarrow$ sparsely **coupled** harmonic oscillators with **heterogeneous** frequencies

Local oscillations and their control

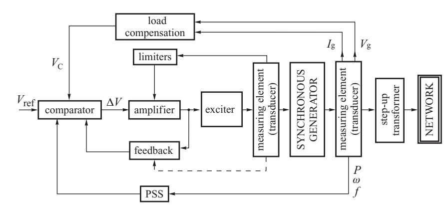
Automatic Voltage Regulator (AVR):



- objective: generator voltage = *const.*

$\Rightarrow$ diminishing damping & sync torque $\frac{\partial P}{\partial \theta}$

$\Rightarrow$ can result in oscillatory instability

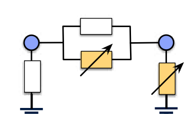


Power System Stabilizer (PSS):

- objective: net damping positive
- typical control design:

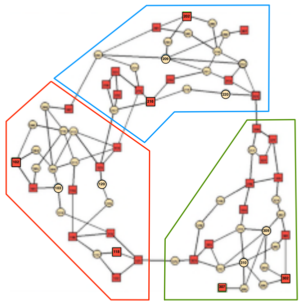
$\rightarrow$ low-pass $\rightarrow$ wash-out $\rightarrow$ lead/lag element $\rightarrow$ gain $\rightarrow$

Flexible AC Transmission Systems (FACTS) or HVDC:

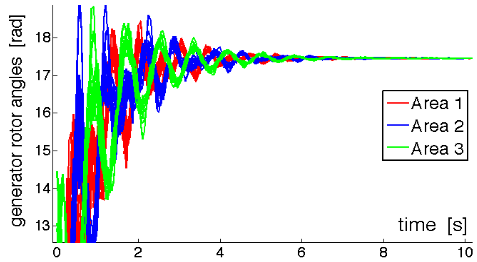


- control by "modulating" transmission line parameters
- either connected in series with a line or as shunt device

Inter-area oscillations in power networks



RTS 96 power network



swing dynamics

Inter-area oscillations are caused by

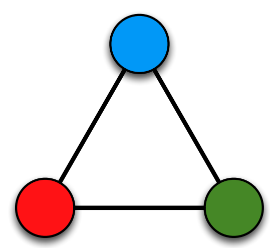
1. **heterogeneity**: fast & slow responses (inertia M_i and damping D_i)
2. **topology**: internally strongly and externally sparsely connected areas
3. **power transfers** between areas: $a_{ij} = B_{ij} E_i^* E_j^* \cos(\theta_i^* - \theta_j^*)$
4. **interaction** of multiple local control loops (e.g., high gain PSSs)

Taxonomy of electro-mechanical oscillations

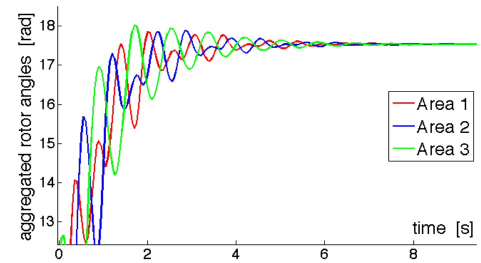
- Synchronous generator = electromech. oscillator ⇒ **local oscillations**:
  - = single generator oscillates relative to the rest of the grid
  - ☹ AVR control induces unstable local oscillations
  - ☺ typically damped by local feedback via Power System Stabilizers
- Power system = complex oscillator network ⇒ **inter-area oscillations**:
  - = groups of generators oscillate relative to each other
  - ☹ poorly tuned local PSSs result in unstable inter-area oscillations
  - ☹ inter-area oscillations are only poorly controllable by local feedback
- Consequences of **recent developments**:
  - ☹ increasing power transfers outpace capacity of transmission system
  - ⟹ ever more lightly damped electromechanical inter-area oscillations
  - ☺ technological opportunities for **wide-area control (WAC)**

Slow Coherency Modeling

Slow coherency and area aggregation



aggregated RTS 96 model



swing dynamics of aggregated model

Aggregate model of lower dimension & with less complexity for

1. analysis and insights into inter-area dynamics [Chow and Kokotovic '85]
2. measurement-based id of equivalent models [Chakrabortty et.al.'10]
3. remedial action schemes [Xu et. al. '11] & wide-area control (later today)

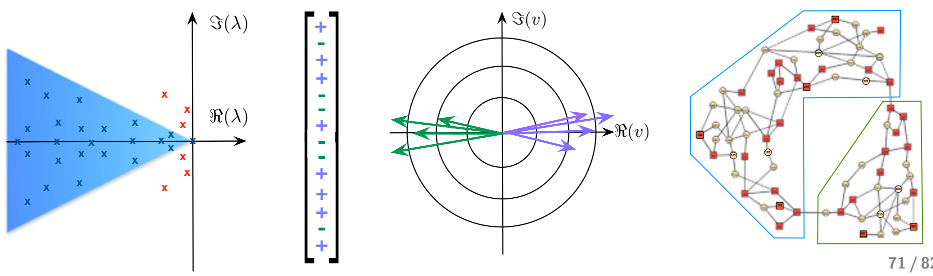
How to find the areas?

classical partitioning ≈ spectral partitioning

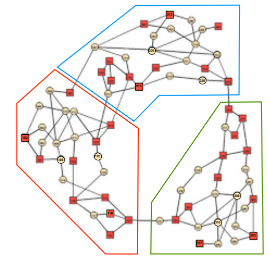
1. construct a linear model $\dot{x} = Ax$ (via, e.g., Power Systems Toolbox)
2. recall solution via eigenvalues λ_i and left/right eigenvectors w_i and v_i:

$$x(t) = \sum_i v_i e^{\lambda_i t} \cdot w_i^T x_0 = \sum_i \{\text{mode } \#i\} \cdot \{\text{contribution from } x_0\}$$

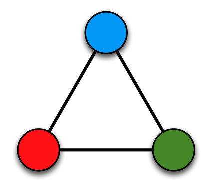
3. look at poorly damped complex conjugate mode pairs
4. look at angle & frequency components of eigenvectors
5. group the generators according to their polarity in eigenvectors



Setup in slow coherency



original model



aggregated model

- r given areas
  (from spectral partition [Chow et al. '85 & '13])
- small **sparsity parameter**:

$$\delta = \frac{\max_\alpha(\Sigma \text{ external connections in area } \alpha)}{\min_\alpha(\Sigma \text{ internal connections in area } \alpha)}$$

- **inter-area dynamics** by center of inertia:

$$y_\alpha = \frac{\sum_{i\in\alpha} M_i \theta_i}{\sum_{i\in\alpha} M_i}, \quad \alpha \in \{1,\dots,r\}$$

- **intra-area dynamics** by area differences:

$$z_{i-1}^\alpha = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1,\dots,r\}$$

Linear transformation & time-scale separation

Swing equation $\Longrightarrow$ singular perturbation standard form

$$M\ddot{\theta} + D\dot{\theta} + L\theta = 0 \quad \Longrightarrow \quad \left\{ \frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix} \right.$$

Slow motion given by center of inertia:

$$y_\alpha = \frac{\sum_{i\in\alpha} M_i \theta_i}{\sum_{i\in\alpha} M_i}, \quad \alpha \in \{1,\dots,r\}$$

Fast motion given by intra-area differences:

$$z_{i-1}^\alpha = \theta_i - \theta_1, \quad i \in \alpha \setminus \{1\}, \alpha \in \{1,\dots,r\}$$

Slow time scale: $t_s = \delta \cdot t \cdot$ "max internal area degree"

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\, \ddot{\varphi} + D_a\, \dot{\varphi} + L_{\text{red}}\, \varphi = 0$$

Properties of aggregated model [D. Romeres, FD, & F. Bullo, '13]

1. $M_a = \begin{bmatrix} \ddots & & \\ & \sum_{i\in\alpha} M_i & \\ & & \ddots \end{bmatrix}$ and $D_a = \begin{bmatrix} \ddots & & \\ & \sum_{i\in\alpha} D_i & \\ & & \ddots \end{bmatrix}$
2. L_{red} = "inter-area Laplacian" + "intra-area contributions"
   = positive semidefinite Laplacian with possibly negative weights

Area aggregation & approximation

- Singular perturbation standard form:

$$\frac{d}{dt_s}\begin{bmatrix} y \\ \dot{y} \\ \sqrt{\delta}\, z \\ \sqrt{\delta}\, \dot{z} \end{bmatrix} = \begin{bmatrix} \ddots & \vdots & \iddots \\ \cdots & A & \cdots \\ \iddots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ z \\ \dot{z} \end{bmatrix}$$

- Aggregated swing equations obtained by $\delta \downarrow 0$:

$$M_a\, \ddot{\varphi} \, + \, D_a\, \dot{\varphi} \, + \, L_{\mathrm{red}}\, \varphi \, = \, 0$$

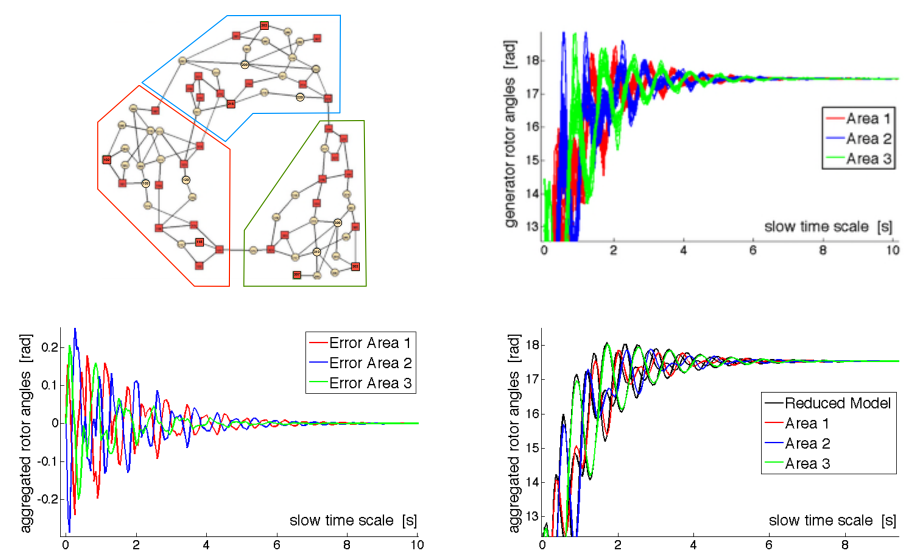
Singular perturbation approximation [D. Romeres, FD, & F. Bullo, '13]

There exist δ^* sufficiently small such that for $\delta \leq \delta^*$ and for all $t > 0$:

$$\begin{bmatrix} y(t_s) \\ \dot{y}(t_s) \end{bmatrix} = \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta})\,, \quad \begin{bmatrix} z(t_s) \\ \dot{z}(t_s) \end{bmatrix} = \tilde{A} \begin{bmatrix} \varphi(t_s) \\ \dot{\varphi}(t_s) \end{bmatrix} + \mathcal{O}(\sqrt{\delta})\,.$$

center of inertia $\approx$ solution of aggregated swing equation

RTS 96 swing dynamics revisited

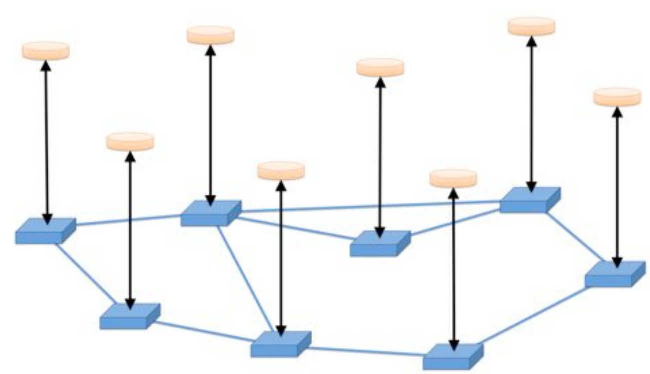


Inter-Area Oscillations & Wide-Area Control

Remedies against electro-mechanical oscillations

conventional control

- Blue layer: interconnected generators

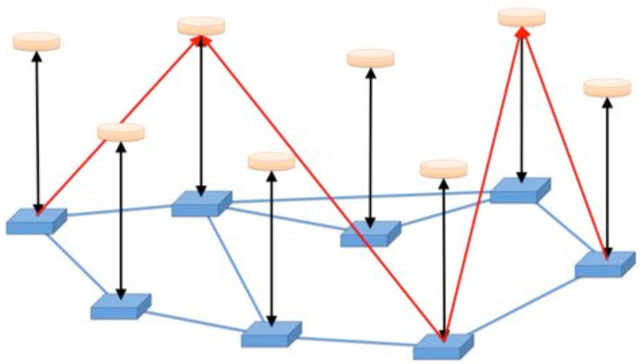


- Fully decentralized control implemented via PSS, HVDC, or FACTS:
  - ☺ effective against local oscillations
  - ☹ ineffective against inter-area oscillations

Remedies against electro-mechanical oscillations

wide-area control

- Blue layer: interconnected generators



- Fully decentralized control
- Distributed wide-area control requires identification of sparse control architecture: actuators, measurements, & communication channels

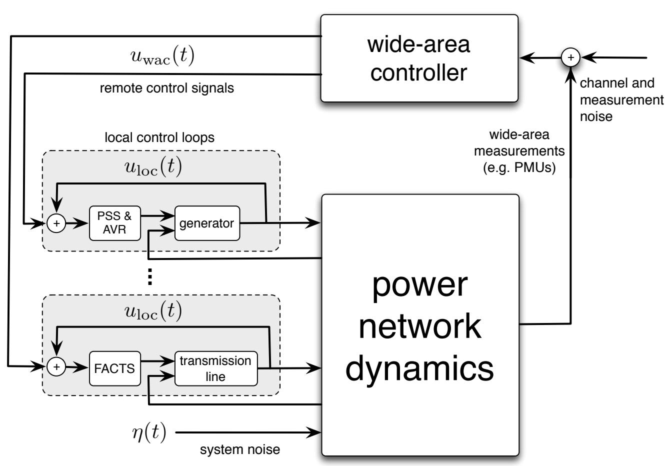
Challenges in wide-area control

- **Objectives:** wide-area control should achieve
  1. optimal closed-loop performance
  2. low control complexity (comm, measurements, & actuation)
- **Problem:** objectives are conflicting
  1. design (optimal) centralized control ⇒ identify control architecture
     - ☹ complete state info & measurements
     - ☹ high communication complexity
  2. identify measurements & control architecture ⇒ design control
     - ☹ decentralized (optimal) control is hard
     - ☹ combinatorial criteria for control channels

Today: simultaneously optimize closed-loop performance & identify sparse control architecture

Setup in Wide-Area Control

1. remote control signals & remote measurements (e.g., PMUs)
2. excitation (PSS & AVR) and power electronics (FACTS) actuators
3. communication backbone network



Optimal Wide-Area Damping Control

Analysis and Design Trade-Offs for Power Network Inter-Area Oscillations

Xiaofan Wu, Florian Dörfler, and Mihailo R. Jovanović

***Abstract*— Conventional analysis and control approaches to inter-area oscillations in bulk power systems are based on a modal perspective. Typically, inter-area oscillations are identified from spatial profiles of poorly damped modes, and they are damped using carefully tuned decentralized controllers. To improve upon the limitations of conventional decentralized strategies, recent efforts aim at distributed wide-area control which involves the communication of remote signals. Here, we introduce a novel approach to the analysis and control of inter-area oscillations. Our framework is based on a stochastically driven system with performance outputs chosen such that the $\mathcal{H}_2$ norm is associated with incoherent inter-area oscillations. We show that an analysis of the output covariance matrix offers new insights relative to modal approaches. Next, we leverage the recently proposed sparsity-promoting optimal control approach to design controllers that use relative angle measurements and simultaneously optimize the closed-loop performance and the control architecture. For the IEEE 39 New England model, we investigate performance trade-offs of different control architectures and show that optimal retuning of decentralized control strategies can effectively guard against inter-areas oscillations.**

damped via decentralized controllers, whose gains are carefully tuned according to root locus criteria [7]–[9].

To improve upon the limitations of decentralized controllers, recent research efforts aim at distributed wide-area control strategies that involve the communication of remote signals, see the surveys [10], [11] and the excellent articles in [12]. The wide-area control signals are typically chosen to maximize modal observability metrics [13], [14], and the control design methods range from root locus criteria to robust and optimal control approaches [15]–[17].

Here, we investigate a novel approach to the analysis and control of inter-area oscillations. Our unifying analysis and control framework is based on a stochastically driven power system model with performance outputs inspired by slow coherency theory [18], [19]. We analyze inter-area oscillations by means of the $\mathcal{H}_2$ norm of this system, as in recent related approaches for interconnected oscillator networks and multi-machine power systems [20]–[22]. We show that an analysis of power spectral density and variance amplification

Talk to conference attendee Xiaofan for the details

there's a lot more to tell,
but I figured this is enough
for two hours of lecture

Conclusions

Introduction

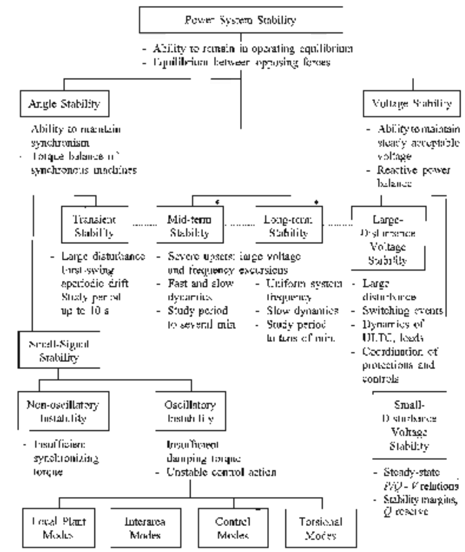
Power Network Modeling

Feasibility, Security, & Stability

Power System Control Hierarchy

Power System Oscillations

Conclusions



Obviously, there is a lot more . . .

I hope I could give you a little insight into a few interesting problems.